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Applications of g*sp-closed sets in topological spaces

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Abstract

The purpose of this paper is to define and study a new class of allied continuous functions and irresolute functions via newly introduced g*sp-closed sets, called g*sp-continuous functions, strongly g*sp-continuous functions, g*sp-irresolute functions, (g,g*sp)-continuous functions , (g*p,g*sp)-continuous functions .Also, we study the concepts of g*sp-connected spaces in this paper.

Keywords: Preopen sets, semi reclosed sets, gsp-closed sets and gp-closed sets, g*p-closed sets, gsp-continuity

1. Introduction

Levine [11] generalized the closed set to generalized closed set (g-closed set) in topology for the first time. Since then it is noticed that some of the weaker forms of closed sets have been generalized. In 1982 and 1986, respectively, A.S.Mashhour et al [12] and D. Andrijevic [1] have defined and studied the concepts of preopen sets and semi preopen sets in topology. In 1995 , 1998 and 2002 , respectively, Dontchev [8] , Noiri et al [16] and M.K.R.S.Veera Kumar[18], have defined and studied the concepts of gsp-closed sets , gp-closed sets and g*p-closed sets in topological spaces . The purpose of this paper is to define and study a new class of allied continuous functions and irresolute functions via newly introduced g*sp-closed sets, called g*sp-continuous functions, strongly g*sp-continuous functions, g*sp-irresolute functions, (g,g*sp)-continuous functions, (g*p,g*sp)-continuous functions . Also, we study the concepts of g*sp-connected spaces in this paper.

2. Preliminaries

Throughout this paper (X,τ) and (Y,σ) (or simply X and Y) always means topological spaces on which no separation axioms are assumed unless explicitly stated. Let A be a subset of space X. We denote the closure of A and the interior of A by Cl(A) and Int(A) respectively.
The following definitions and results are useful in the sequel.

2.1 Definition [9]

A subset A of a space X is said to be

(i) Preopen [12] if $A \subset$ Int Cl(A).
(ii) Semiopen [10] if $A \subset$ ClInt(A).
(iii) Semipreopen [1] if $A \subset$ Cl Int Cl(A).

The complement of a preopen (resp. semiopen , semipreopen) set of a space X is called preclosed [9] (resp. semiclosed [5] ,semipreclosed [1]).

2.2 Definition [1]

The union of all semipreopen sets contained in A is called the semipreinterior of A and is denoted by spInt(A). pInt(A)[13] and sInt(A)[7] can be similarly defined.

2.3 Definition [1]

The intersection of all semipreclosed sets containing A is called the semipreclosure of A and is denoted by spCl(A). pCl(A)[9] and sCl(A)[6] can be similarly defined.

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2.4 Definition

A subset A of a space X is called:
(i)generalized closed set (in brief , g-closed) set [11] if $Cl(A)\subset U$ whenever $A \subset U$ and U is open in X .

(i) Generalized semiclosed (in brief, gs-closed) set [3] if $Cl(A)\subset U$ whenever $A \subset U$ and U is open in X .
(ii) Generalized semipreclosed (in brief, gsp-closed) set [8] if $spCl(A)\subset U$ whenever $A \subset U$ and U is open in X
(iii) Generalized preclosed (in brief, gp-closed) set [16] if $pCl(A)\subset U$ whenever $A \subset U$ and U is open in X .
(iv) g*-closed set [17] if $Cl(A) \subset U$ whenever $A \subset U$ and U is g-open in X .
(v) g*-preclosed (in brief, g*p-closed) set [18] if $pCl(A)\subset U$ whenever $A \subset U$ and U is g-open in X.

2.5 Definition

A function f: X →Y is said to be

(i) Pre continuous [12] if the inverse image of each open set of Y is preopen in X.
(ii) Semipre continuous [14] if the inverse image of each open set of Y is semi preopen in X.
(iii) gp-continuous [2] if the inverse image of each open set of Y is gp-open in X .
(iv) g*p-continuous [18] if the inverse image of each open set of Y is g*p-open in X .
(v) gsp-continuous [8] if the inverse image of each open set of Y is gsp-open in X .
(vi) g-continuous [4] if the inverse image of each open set of Y is g-open in X.
(vii) gc-iresolute [4] if the inverse image of each g-open set of Y is g-open in X.

3. g*sp- continuous and g*sp-irresolute functions

We, recall the following.

3.1 Definition [15]

A subset A of a space X is said to be g*-semipreclosed [in brief, g*sp-closed] set if $spCl(A) \subset U$ henever $A\subset U$ and U is g-open in X.
The complement of a g*sp-closed set is called g*sp-open set.
The family of all g*sp-open sets of X is denoted by G*SPO(X).

3.2 Lemma [15]

Let X be a space, then,

(i) Every closed set is g*sp-closed set.
(ii) Every g-closed set is g*sp-closed set.
(iii) Every preclosed set is g*sp-closed set.
(iv) Every semipreclosed set is g*sp-closed set.
(v) Every g*-closed set is g*sp-closed set.
(vi) Every gs-closed set is g*sp-closed set.
(vii) Every g*sp-closed set is gsp-closed set.
(viii) Every g*p-closed set is g*sp-closed set.
(ix) Every gp-closed set is g*sp-closed set.

We define the following.

3.3 Definition

A function f: X →Y is said to be g*sp-continuous if inverse image of each closed set of Y is g*sp -closed in X .
In view of Lemma-3.2 above, We have the following.

3.4 Lemma

Let f : X → Y be a function . Then,
(i) If f is continuous (and hence pre-continuous, semi pre-continuous) function, then it is g*sp-continuous.
(ii) If f is g*sp-continuous function, then it is gsp-continuous.
(iii) If f is g*p-continuous function, then it is g*sp-continuous.
(iv) If f is gp-continuous function, then it is g*sp-continuous.
(v) If f is g*p-continuous function, then it is g*sp-continuous.

3.5 Definition

A function f: X →Y is said to be strongly g*sp-continuous if inverse image of each g*sp - closed set of Y is closed in X

3.6 Definition

A function f: X →Y is said to be g*sp-irresolute if inverse image of each g*sp -closed set of Y is g*sp -closed in X

3.7 Theorem

Every g*sp-irresolute function is g*sp-continuous

Proof: Suppose f: X→Y is g*sp-irresolute. Let V be any closed subset of Y. Then V is g*sp-closed in Y , lemma -3.2 . Since f is g*sp-irresolute, $f^{-1}(V)$ is g*sp-closed in X. This proves the theorem.

3.8 Theorem

Let f: X→Y and g: Y→Z be two g*sp-irresolute functions, then gof is also g*sp-irresolute function.

Proof: Obvious

3.9 Theorem

Let f: X→Y be a function. Then the following are equivalent.

(i) f is g*sp-continuous
(ii) The inverse image of each open set Y is g*sp-open in X
(iii) The inverse image of each closed set in Y is g*sp-closed in X

Proof: (i)⟹(ii): Let G be open in Y. Then Y-G is closed in Y. By (i) f^{-1}(Y-G) is g*sp-closed in X. But f^{-1}(Y-G) = X -- f^{-1}(G) which is g*sp-closed in X. Therefore f^{-1}(G) is g*sp-open in X. (ii)⟹(iii) and (iii)⟹(i) follow easily.

We, recall the following

3.10 Definition[15]

The intersection of all g*sp-closed sets containing A is called the g*sp-closure of A and is denoted g*spCl(A)

3.11 Lemma[15]

Let x∈X, then x∈ g*spCl(A) if and only if V∩A≠ ϕ for every g*sp-open set V containing x .

We, prove the following

3.12 Theorem

If a function f: X→Y is g*sp-continuous then f(g*spCl(A)) ⊆ Cl(f(A)) for every subset A of X.

Proof: Let f: X→Y be g*sp-continuous. Let A⊆X. Then Cl (f(A)) is closed in Y. Since f is g*sp -continuous, f^{-1}(Cl (A)) is g*sp-closed in X.Suppose y ∈ f(x), x ∈ g*spCl(A) . Let G be an open set containing y ∈ f(x). Since f is g*sp-continuous. Then, f^{-1}(G) is g*sp-open containing x so that f^{-1}(G)∩A ≠ ∅ by Lemma 3.11. Therefore f^{-1}(f^{-1}(G) ∩A)≠ ϕ which implies f(f^{-1}(G)∩f(A))≠ ϕ.Since f(f^{-1}(G)) ⊆ G, G∩f(A)≠ ϕ.This proves that y∈Cl(f(A)) that implies f(g*sp Cl(A)) ⊆ Cl(f(A)).

3.13 Theorem

If a function f:X→Y is g*sp-irresolute then f(g*spCl(A)) ⊆ g*spCl(f(A)) for every subset A of X.

Proof: Similar to Th.3.11.

We, recall the following.

3.14 Lemma [15]

A subset A of space X is called g*sp-open set if U⊆ spInt(A) whenever U⊆A and U is g-closed set in X

We, recall the following

3.15 Definition[15]

The union of all g*sp-open sets which contained in A is called the g*sp-interior of A and is denoted by g*spInt(A)

3.16 Theorem

Let f: X→Y be g*sp-continuous and g: Y→Z be continuous, then gof: X →Z be g*sp-continuous.

Proof: Let V be any open subset of Z. Then g^{-1}(V) is open in Y, since g is continuous function. Again, f is g*sp- continuous and g^{-1}(V) is open set in Y then $f^{-1}(g^{-1}(V)) = (gof)^{-1}(V)$ is g*sp -open in X. This shows that gof is g*sp-continuous.

3.17 Theorem

Let f: X→Y be g*sp-continuous and g: Y→Z be strongly g*sp -continuous, then gof: X →Z be g*sp-irresolute.

Proof: Let V be any g*sp -open subset of Z. Then g^{-1}(V) is open in Y,since g is strongly g*sp -continuous function. Again, f is g*sp- continuous and g^{-1}(V) is open set in Y then $f^{-1}(g^{-1}(V)) = (gof)^{-1}(V)$ is g*sp-open in X. This shows that gof is g*sp-irresolute.

We, define the following

3.18 Definition

A function f: X→Y is called contra g*sp - continuous if f^{-1}(V) is g*sp-closed in X for each open set V in Y.

3.19 Theorem

Let f: X→Y be g*sp-continuous and g: Y→Z be contra-continuous, then gof: X →Z be contra g*sp-continuous.

Proof: Obvious.

We, define the following

3.20 Definition

A set U⊂X is said to be a g*sp-neighbourhood of a point x∈X if and only if there exists a g*sp-open set A in X such that x∈A⊂U.

3.21 Theorem

The following statement are equivalent for a function f: X→Y:

(i) f is strongly g*sp-continuous.

(ii) For each point x of X and each g*sp-neighbourhood V of f(x),there exist a open-neighbourhood U of x such that f(U) ⊆V

(iii) For each x in X and each V∈ G*SPO(f(x)), there exists an open set U in X such that f(U) ⊆V

Proof: (i)⟹(ii): Assume x∈ X and V is g*sp-open set in Y containing f(x). Since, f is strongly g*sp-continuous and let $U=f^{-1}(V)$ be a open set in X containing x and hence f (U) = $f(f^{-1}(V))\subset V$.

(ii)⟹(iii): Assume that V⊂Y is a g*sp-open set containing f(x), Then by (ii) there exists a open set U such that $x\in U \subset f^{-1}(V)$. Therefore, $x\in f^{-1}(V) \subset Cl(f^{-1}(V))$. This shows that Cl $(f^{-1}(V))$ is a open-neighborhood of x.

(iii)⟹(i): Let V be a g*sp-open set in Y, then $Cl(f^{-1}(V))$ is a open neighborhood of each $x \in f^{-1}(V)$. Thus, for each x is a interior point of Cl $(f^{-1}(V))$ which implies that $f^{-1}(V) \subset U$. Therefore, $f^{-1}(V)$ is an open set in X and hence f is a strongly g*sp-continuous function.

We, define the following.

3.22 Definition

A function f: X→Y is called (g, g*sp) -continuous if the inverse image of each g-open set of Y is g*sp-open in X.

Clearly, every (g, g*sp)-continuous function is g*sp-continuous function, since every open set is g-open set.

3.23 Theorem

Let f: X→Y be (g, g*sp)-continuous function and g: Y→Z be g-continuous then gof: X→Z is g*sp -continuous function.

Proof: Obvious.

3.24 Theorem

Let f: X→Y be (g, g*sp)-continuous function and g :Y→Z be gc-irresolute then gof :X→Z is (g,g*sp) -continuous function.

Proof: Obvious.

We, define the following.

3.25 Definition

A function f: X→Y is called (g*p , g*sp)-continuous if the inverse image of each g*p-open set of Y is g*sp-open in X.

3.26 Definition

A function f : X→Y is called strongly g*p-continuous if the inverse image of each g*p-open set of Y is open in X.

The routine proofs of the following are omitted.

3.27 Theorem

Let f: X→Y be strongly-g*sp-continuous function and g: Y→Z be g*sp-irresolute then gof: X→Z is strongly g*sp-continuous function.

3.28 Theorem

Let f: X→Y be strongly-g*sp-continuous function and g: Y→Z be (g*p, g*sp)-continuous then gof: X→Z is strongly g*p-continuous function.

Next, we recall the following.

3.29 Definition[15]

A space X is said to be a

(i) T_{sp}*-space if every g*sp-closed set in it is closed.

(ii) *T_{sp} – space if every g*sp-closed set in it is semipro closed.

(iii) T_{gsp}*-space if every gsp-closed set in it is g*sp-closed.

(iv) **T_{gsp}-space if every g*sp-closed set in it is g*p-closed.

(v) T_{gp}**-space if every gp-closed set in it is g*sp-closed.

We, state the following.

3.30 Lemma

Let f: X→Y be an gsp-continuous function with X as a T^*_{gsp}-space, then f is g*sp-continuous function.

We, define the following.

3.31 Definition

A space X is said to be g*sp-connected if X cannot be written as the disjoint union of two nonempty g*sp-open sets in X.

3.32 Definition

A space X is said to be g*p-connected if X cannot be written as the disjoint union of two nonempty g*p-open sets in X.

3.33 Definition

A space X is said to be gsp-connected if X cannot be written as the disjoint union of two nonempty gsp-open sets in X.

3.34 Definition

A function f: X→Y is (gsp,g*sp)-continuous if the inverse image of each gsp-open set of Y is g*sp-open in X.

We, give the following.

3.35 Lemma

For a space X, the following are equivalent:
(i) X is g*sp-connected.
(ii) X and ∅ are the only subsets of X which are both g*sp-open and g*sp-closed.
(iii) Each g*sp-continuous function of X into some discrete space Y with at least two points is a constant function.

Proof: Obvious.

3.36 Theorem

Let f: X→Y be a function
(i) If X is g*sp-connected and if f is g*sp-continuous, surjective, then Y is connected.
(ii) If X is g*sp-connected and if f is g*sp-irresolute, surjective, then Y is g*sp-connected.

Proof:

(i) Let X be g*sp-connected and f be g*sp-continuous, surjective. Suppose Y is disconnected. Then Y = A∪B, where A and B are nonempty open subset of Y. Since f is g*sp-continuous surjective then, X= $f^{-1}(A) \cup f^{-1}(B)$ where $f^{-1}(A)$ and $f^{-1}(B)$ are disjoint nonempty g*sp-open subsets of X. This contradicts the fact that X is g*sp-connected. Therefore Y is connected. This proves (i).
(ii) Let X be g*sp-connected and f be g*sp-irresolute surjective. Suppose Y is not g*sp-connected. Then Y = A∪B, where A and B are disjoint nonempty g*sp-open subsets of Y. Since f is g*sp-irresolute surjective, then X= $f^{-1}(A) \cup f^{-1}(B)$ where $f^{-1}(A)$ and $f^{-1}(B)$ are disjoint nonempty g*sp-open subsets of X. This implies X is not g*sp-connected, which is contradiction. Therefore Y is g*sp-connected. This prove (ii).

3.37 Theorem

Let f: X→Y be a function
(i) If X is g*sp-connected and if f is (g*p,g*sp)-continuous, surjective, then Y is g*p-connected.
(ii) If X is g*sp-connected and if f is (gsp,g*sp)-continuous, surjective, then Y is gsp-connected.

Proof: Obvious.

4. Conclusion

In this paper, a new class of generalized closed sets, namely g*sp-closed sets in topological spaces is defined and studied analogous to the existing class of sets called g*p-closed due to M.K.R.S. Veera Kumar [18]. Applying g*sp-closed sets, we introduce and study some classes of spaces, namely T_{sp}*-spaces, *T_{sp}– spaces, T_{gsp}*-spaces,**T_{gsp}-spaces and T_{gp}**-spaces in [15].In view of these spaces , as recalled in Definition 3.30, we conclude the usefulness of these spaces in the following.

4.1 Theorem

Let f: X→Y be a g*sp-continuous function. Then,
(i) If X is a T_c*-space, then f is continuous function.
(ii) If X is a T^*_{pc}-space, then f is pre-continuous function.
(iii) If X is a *T_{sp}-space, then f is semipro continuous function.
(iv) If X is a **T_{gsp}-space, then f is g*p-continuous function.
(v) If X is a T_{gp}**-space, then f is gp-continuous function.

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